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Short communication

Equivalent circuit components of nickel–cadmium battery at different states of charge

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ABSTRACT

Equations that describe the voltage variations with time of a rechargeable battery during charging and discharging were used to determine the component values of the equivalent circuit of nickel–cadmium batteries under different states of charge (SOC). The equivalent circuit of the battery was described as an ideal voltage source in series with a resistor and the parallel combination of a resistor and a capacitor. The battery model used different values of resistance and capacitance, in the parallel combination, during the different phases of the discharge-rest–charge-rest sequence. The results show that the series resistance is approximately constant with variations in the SOC while the resistor in the parallel *RC* circuit increases as the SOC decreases. For the discharge and charge phases the capacitor value increased and decreased, respectively, as the SOC decreased. The value of the resistor or capacitor in the parallel *RC* circuit is an indicator of the battery SOC.

1. Introduction

The engineering design and performance evaluation of battery dependent systems require an electrical equivalent circuit for the battery. The variation of the equivalent circuit components of nickel–cadmium batteries as a function of the battery state of charge (SOC) is given in this paper. Impedance methods have been used in the past to model the state of charge of batteries [1,2]. Other methods use coulomb counting [3] or the measurement of the open circuit voltage to estimate the SOC [4]. The present work uses a transient method to model the battery parameters at different SOC.

The most basic circuit of the battery is a voltage source in series with the internal resistance. The electrochemical processes of rechargeable batteries [5,6] lead to a second, and more precise, equivalent circuit comprising a voltage source in series with a resistor and a parallel combination of a resistor and a capacitor [7–11]; the voltage–resistance–capacitance (VRC) model. The latter is used to model the battery under conditions of constant state of charge and constant temperature. This model is valid for discharge times of the order of seconds. This model was used in the present work together with a battery measurement method based on transient analysis.

The VRC model was used in the present work in a transient measurement method that uses the discharge-rest–charge-rest sequence of a rechargeable battery with different values for the resistor and capacitor, in the parallel combination of *R* and *C*, for the charge and discharge phases and for the intervening rest periods. Equations for the temporal variation of the battery voltage during transients, including boundaries at transitions between transients, were used to calculate the equivalent circuit components for a nickel–cadmium battery at three values of SOC.

2. Theory

The equivalent circuit of the battery appears to the left of terminals *a* and *b* in Fig. 1 where $v(t)$ is the battery voltage. It is a series combination of the open circuit battery voltage V_{oc}, a resistor, and the equivalent of the parallel combination of a resistor and a capacitor.

Fig. 1(i) shows the battery equivalent circuit during the discharge phase. R_{ex} is the external resistor. Fig. 1(ii) shows the battery equivalent circuit in the rest period that follows the discharge phase.

Fig. 2(i) shows the battery circuit during the charge phase. V_g is the charging voltage. The charge phase is followed by a rest period represented by Fig. 2(ii). The same battery model applies in the charge phase as in the discharge one but now with different values for the components in the parallel circuit. The capacitor value is C_2 for both the charge and the subsequent rest period, while R_3 and R_4 are the resistors in parallel with the capacitor in the charge and rest periods, respectively. The series resistor *R* stays the same as in the discharge-rest case. Since $V_g > V_{oc}$ the voltage source V_g acts as a generator. The current $i(t)$ flows into terminal *a* and ultimately into

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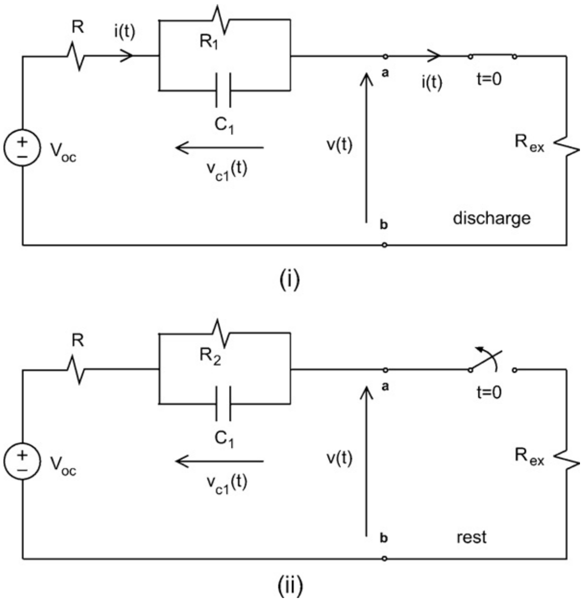

Fig. 1. The circuit for (i) the discharge period and (ii) the subsequent rest phase of the battery. $v(t)$ is the battery voltage and R_{ex} is the external load resistor. V_{oc} is the open circuit voltage of the battery. R is the series resistance in the battery equivalent circuit.

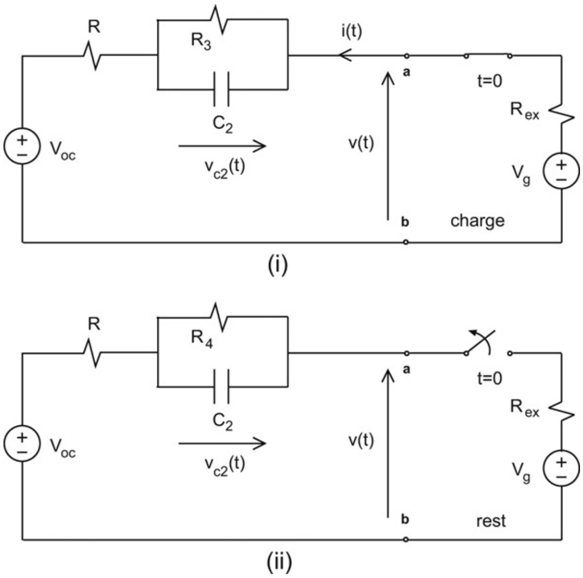

Fig. 2. The equivalent circuit for the battery in (i) the charging phase followed by (ii) the resting period. V_g is the external charging voltage. $V_g > V_{oc}$.

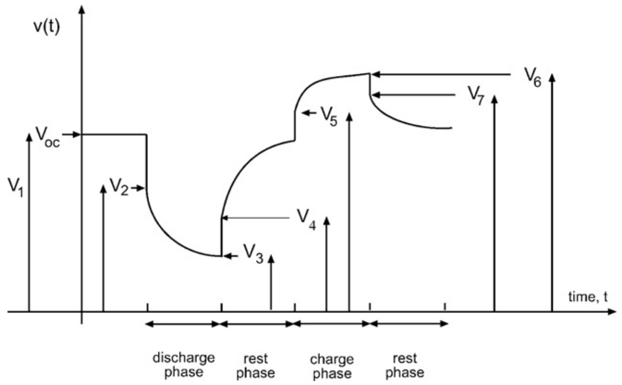

Fig. 3. The battery terminal voltage $v(t)$ as a function of time during the discharge and rest phases, followed by the charge and rest phases showing the voltage levels V_1–V_7.

the positive terminal of V_{oc} as a charging current to the battery. The source V_{oc} now acts as the energy absorber.

Transients are involved during the charging and discharging of batteries. Abrupt changes are recorded in the voltage across the terminals of the battery as the switch is closed or opened at the start of these transients. These boundary value changes in voltage as well as voltage levels in the intervening transients are used here to calculate circuit parameters. The temporal variation of the voltage across the battery terminals during the discharge-rest–charge-rest sequence appears in Fig. 3. The equations for the variation in the battery voltage $v(t)$ during the discharge-rest–charge-rest sequence were derived earlier [12]. These equations correspond to the voltage levels V_1–V_7 that are shown in Fig. 3 and are relevant to the measurement technique used here. These equations are

$$V_1 = V_{oc} \tag{1}$$

$$V_2 = \left(\frac{R_{ex}}{R + R_{ex}}\right) V_{oc} \tag{2}$$

$$V_3 = \left(\frac{R_{ex}}{R + R_1 + R_{ex}}\right) V_{oc} \tag{3}$$

$$V_4 = \left(\frac{R + R_{ex}}{R + R_1 + R_{ex}}\right) V_{oc} \tag{4}$$

$$V_5 = V_g - \left(\frac{R_{ex}}{R + R_{ex}}\right)(V_g - V_{oc}) \tag{5}$$

$$V_6 = V_g - \left(\frac{R_{ex}}{R + R_3 + R_{ex}}\right)(V_g - V_{oc}) \tag{6}$$

$$V_7 = V_{oc} + \left(\frac{R_3}{R + R_3 + R_{ex}}\right)(V_g - V_{oc}) \tag{7}$$

Gradients of the battery voltage at the start of various transients were used, in addition to the equations shown above, for the calculations of the battery circuit parameters. The respective gradients at the start of the discharge phase and at the start of the subsequent rest period are given by

$$\left(\frac{dv}{dt}\right)_d = -\frac{R_{ex}V_{oc}}{(R + R_{ex})^2 C_1} \tag{8}$$

$$\left(\frac{dv}{dt}\right)_{r1} = \frac{R_1 V_{oc}}{(R + R_1 + R_{ex})R_2 C_1} \tag{9}$$

The respective gradients at the start of the charge phase and at the start of the subsequent rest period are given by

$$\left(\frac{dv}{dt}\right)_c = \frac{R_{ex}(V_g - V_{oc})}{(R + R_{ex})^2 C_2} \tag{10}$$

$$\left(\frac{dv}{dt}\right)_{r2} = -\frac{R_3(V_g - V_{oc})}{(R + R_3 + R_{ex})R_4 C_2} \tag{11}$$

The calculation of certain circuit parameters is easier if the change in battery voltage at boundaries between phases rather than absolute values of battery voltages is used. The equations giving these abrupt voltage changes are

$$V_{oc} - V_2 = \left(\frac{R}{R + R_{ex}}\right) V_{oc} \tag{12}$$

$$V_4 - V_3 = \left(\frac{R}{R + R_1 + R_{ex}}\right) V_{oc} \tag{13}$$

$$V_5 - V_{oc} = \left(\frac{R}{R + R_{ex}}\right)(V_g - V_{oc}) \tag{14}$$

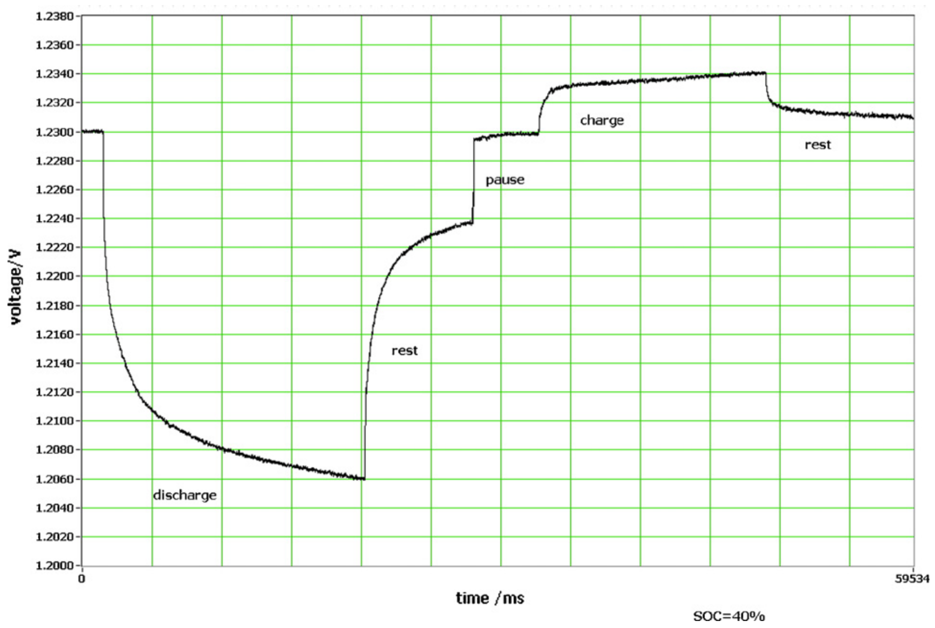

Fig. 4. The temporal variation of the battery voltage $v(t)$ in the discharge-rest–charge-rest characteristics of a NiCad battery at a state of charge of 40%.

$$V_6 - V_7 = \left(\frac{R}{R + R_3 + R_{ex}}\right)(V_g - V_{oc}) \tag{15}$$

The above voltages, voltage differences and slopes are obtained from experimentally measured curves of the temporal variation of discharge-rest–charge-rest characteristic of the battery. To calculate the equivalent circuit components of the battery these values of voltages and slopes are then substituted in the equations given below, where from Eq. (2)

$$R = R_{ex}\left(\frac{V_{oc}}{V_2} - 1\right) \tag{16}$$

from Eq. (5)

$$R = R_{ex}\left(\frac{V_g - V_{oc}}{V_g - V_5} - 1\right) \tag{17}$$

from Eq. (8)

$$C_1 = -\frac{R_{ex}V_{oc}}{(R + R_{ex})^2(dv/dt)_d} \tag{18}$$

from Eq. (9)

$$R_2 = \frac{R_1 V_{oc}}{(R + R_1 + R_{ex})C_1(dv/dt)_{r1}} \tag{19}$$

from Eq. (10)

$$C_2 = \frac{R_{ex}(V_g - V_{oc})}{(R + R_{ex})^2(dv/dt)_c} \tag{20}$$

from Eq. (11)

$$R_4 = -\frac{R_3(V_g - V_{oc})}{(R + R_3 + R_{ex})C_2(dv/dt)_{r2}} \tag{21}$$

while the voltage differences $(V_{oc} - V_2)$, $(V_4 - V_3)$, $(V_5 - V_{oc})$ and $(V_6 - V_7)$ are substituted in the equations below where from Eq. (12)

$$R = \frac{R_{ex}}{((V_{oc}/V_{oc} - V_2) - 1)} \tag{22}$$

from Eq. (13)

$$R_1 = R\left(\frac{V_{oc}}{V_4 - V_3} - 1\right) - R_{ex} \tag{23}$$

from Eq. (14)

$$R = \frac{R_{ex}}{((V_g - V_{oc}/V_5 - V_{oc}) - 1)} \tag{24}$$

and from Eq. (15)

$$R_3 = R\left(\frac{V_g - V_{oc}}{V_6 - V_7} - 1\right) - R_{ex} \tag{25}$$

3. Results and discussion

Nickel–cadmium batteries rated at 1.2 V, and a 10-h capacity of 1300 mAh were examined. The discharge-rest–charge-rest curve of a fully charged battery was first obtained. The battery was then discharged to 70% SOC and then to 40% SOC and the discharge-rest–charge-rest curve at each SOC was recorded. Each curve was a plot of the battery voltage $v(t)$ as a function of time and was recorded with the battery at room temperature. Fig. 4 shows the discharge-rest–charge-rest curve of a battery at 40% SOC.

The charging voltage V_g during the charge phase was 1.456 V for all experiments. The external load resistor R_{ex} was 11.7 Ω. The open circuit battery voltage V_{oc} was 1.230 V, 1.238 V and 1.361 V at 40%, 70% and 100% SOC, respectively. Signal acquisition was interrupted for 1.75 h during the rest phase that followed the discharge one, (at the point marked "pause" in Fig. 4, for example) to allow the battery to recover and reach its initial open circuit voltage. Signal acquisition was then resumed to proceed with the recording of the charge-rest phases.

Circuit components could be calculated to within ±0.02. The component values of the equivalent circuit of the battery are given in Table 1. The series resistor R is approximately constant while R_1, R_2, R_3 and R_4 increase as the SOC decreases. Also, C_1 increases while C_2 decreases as the SOC decreases. The same trends were seen in the other NiCad batteries with the same rating that were examined by the author.

The changes in resistance and capacitance with changes in SOC provide a useful means of battery evaluation. For example, from Table 1, the change in R_1 is from 0.076 Ω to 0.194 Ω, in R_2 from 0.097 Ω to 0.277 Ω, in R_3 from 0.861 Ω to 6.32 Ω and in R_4 from 0.853 Ω to 4.066 Ω as the respective SOC changes from 100% to 40%. The changes in capacitance are from 4.861 F to 6.545 F for C_1

Table 1
Component values of the equivalent circuit of the battery at states of charge of 40%, 70% and 100%.

SOC	40%	70%	100%
R	0.060 Ω	0.058 Ω	0.053 Ω
R_1	0.194 Ω	0.167 Ω	0.076 Ω
R_2	0.277 Ω	0.246 Ω	0.097 Ω
C_1	6.545 F	5.186 F	4.863 F
R_3	6.32 Ω	5.11 Ω	0.861 Ω
R_4	4.066 Ω	3.664 Ω	0.853 Ω
C_2	3.134 F	3.536 F	4.244 F

and from 4.244 F to 3.134 F for C_2 as the SOC decreases from 100% to 40%. Therefore, the value of the resistor or capacitor in the parallel *RC* circuit of the battery equivalent can be used as an indicator of the battery SOC.

4. Conclusions

Equations that describe the voltage variations with time of a rechargeable battery during charging and discharging, and in the boundary regions between these phases, were used to determine the component values of the equivalent circuit of nickel–cadmium batteries under different states of charge. The results show that the series resistance is approximately constant with variations in the state of charge while the resistor in the parallel resistor–capacitor circuit increases as the state of charge decreases. For the discharge and charge phases the capacitor value increased and decreased, respectively, as the SOC decreased. The value of the resistor or capacitor in the parallel resistor–capacitor circuit is an indicator of the battery SOC.

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